**Abstract**

1. **Introduction**

Ordinary differential equations (ODEs) serve as fundamental instruments in the mathematical modeling and analytical study of numerous scientific and engineering systems. They naturally emerge in diverse applications, including but not limited to fluid dynamics, chemical reaction kinetics, population dynamics, and structural analysis. The inherent complexity of ODEs presents significant challenges in their solution, as numerous cases do not yield closed-form solutions, necessitating the utilization of numerical or approximation techniques. Established numerical methodologies, including the Runge-Kutta technique, finite difference approach, and shooting method, have historically been employed to tackle these challenges. However, their drawbacks—such as considerable computational demands and the inability to produce closed-form solutions—have motivated the investigation of alternative strategies for ODE resolution.

The introduction of artificial neural networks (ANNs) has facilitated novel methodologies for the numerical resolution of ordinary differential equations (ODEs) by recontextualizing the problem as an optimization framework. Preliminary investigations have demonstrated the efficacy of multilayer perceptrons (MLPs) in approximating solutions to both initial value problems (IVPs) and boundary value problems (BVPs). These neural network-based approaches present significant advantages over traditional numerical methods. Specifically, ANNs can produce analytic solutions that eliminate the necessity for interpolation across discretized computational intervals, thus providing enhanced adaptability in addressing both IVPs and BVPs. However, early iterations of ANN models encountered impediments, including a pronounced vulnerability to convergence at local minima and suboptimal rates of convergence.

In order to address the limitations of traditional artificial neural networks (ANNs), advanced architectures have been introduced, including Radial Basis Function Neural Networks (RBFNNs), Wavelet Neural Networks (WNNs), and Functional Link Neural Networks (FLNNs). These specialized architectures exhibit accelerated convergence rates and improved precision in the approximation of solutions for intricate differential equations. Notably, WNNs have attracted considerable interest due to their localized activation functions, which facilitate compact network designs and expedite the learning process while maintaining the universal approximation capability characteristic of neural networks. Furthermore, the implementation of sophisticated training methodologies, such as Extreme Learning Machines (ELM) and metaheuristic optimization techniques, including Particle Swarm Optimization (PSO), has substantially enhanced both the efficiency and accuracy of these neural network models.

Building upon this foundational principle, the Kolmogorov-Arnold Network (KAN) architecture presents a novel and robust framework specifically engineered for function approximation, demonstrating considerable potential for addressing ordinary differential equations (ODEs). The KAN model is fundamentally grounded in the Kolmogorov-Arnold representation theorem, which asserts that any continuous multivariate function can be expressed as a finite sum of univariate functions. This intrinsic universality renders KAN particularly adept at approximating intricate mathematical models, including those characterized by ODEs. By capitalizing on KAN’s systematic approach to function decomposition, researchers seek to transcend the limitations inherent in existing neural network architectures when tackling higher-order differential equations.

The principal objective of this research is to employ the KAN architecture for the approximation of solutions to first- and second-order ordinary differential equations (ODEs). This examination signifies a substantial advancement in the integration of sophisticated machine learning methodologies within computational mathematics. In contrast to conventional artificial neural network (ANN)-based approaches, the KAN framework intrinsically facilitates dimensionality reduction of the problem space, thereby enhancing the efficiency of the approximation process. Additionally, its distinctive structure allows the network to attain elevated accuracy with a reduced number of parameters, thereby decreasing computational overhead while upholding precision.

The rationale for implementing KAN in this framework arises from its capabilities to effectively tackle critical challenges associated with the resolution of ordinary differential equations (ODEs). Specifically, higher-order ODEs frequently present intricate boundary conditions and exhibit nonlinear dynamics that pose difficulties for conventional numerical techniques. The intrinsic adaptability of KAN, coupled with its competence in representing these complexities, positions it as a viable candidate for addressing such challenges. Furthermore, KAN’s modular architecture promotes the incorporation of sophisticated optimization algorithms, thereby augmenting its efficacy in the resolution of ODEs.

Recent investigations emphasize the efficacy of neural network architectures in the resolution of differential equations. Specifically, wavelet neural networks, when enhanced through sophisticated optimization techniques such as the butterfly optimization algorithm, exhibit superior capabilities in approximating solutions to partial differential equations (PDEs). Additionally, radial basis function neural networks (RBFNNs) trained via extreme learning methodologies demonstrate rapid convergence rates and high accuracy regarding fractional differential equations. These advancements signify the increasing significance of neural network frameworks in the progression of computational mathematics.

Notwithstanding the advancements made in this field, significant deficiencies persist in the literature concerning the application of Kolmogorov-Arnold networks (KAN) to ordinary differential equations (ODEs). Although the Kolmogorov-Arnold theorem offers a theoretical framework for function approximation, its practical deployment for the resolution of ODEs remains insufficiently investigated. This research endeavors to fill this lacuna by executing a thorough assessment of KAN's effectiveness in solving both first- and second-order ODEs. Through methodical experimentation, this study aims to validate KAN as a robust and efficient methodology for function approximation specifically within the context of differential equations.

The implications of this research transcend the direct utilization of KAN in the context of ordinary differential equations (ODEs). By establishing its efficacy as a versatile function approximator, this investigation enriches the field of computational mathematics and neural network-based modeling. The findings derived from this study are anticipated to guide the advancement of next-generation computational methodologies adept at solving intricate scientific and engineering challenges, consequently, the Kolmogorov-Arnold Network (KAN) constitutes a significant progression in the application of machine learning techniques for the resolution of differential equations. Its distinctive architectural framework and theoretical foundations establish it as a formidable alternative to prevailing artificial neural network (ANN) methodologies. This research aims to enhance the current capabilities of neural network-based approaches in addressing first- and second-order ordinary differential equations (ODEs) by leveraging KAN, thereby facilitating advancements in computational mathematics and related fields.

1. **Material and Methods**
   1. **Problems**
      1. **Example 1**
      2. **Example 2**
   2. **Algorithm/Architecture(KAN)**
   3. **Analysis** 
      1. **Model performance and generalizability**
         1. **Experiment setup for algorithm**
      2. **Evaluation**
         1. **Evaluation for algorithm 1**
2. **Results** 
   1. **Model performance and generalizability**
      1. **Model performance and generalizability**
      2. **Evaluation**
      3. **Full evaluation**
3. **Discussion**

**Acknowledgements (if there are any)**

**References**